Horizontal and Vertical Components of a Vector

Vectors

Quantities such as area, temperature, and time have magnitude only and are completely described by a single real number with the appropriate units (square feet, degrees, or minutes). These quantities are called scalar quantities. Some quantities, such as velocity, acceleration, or force, require both magnitude and direction to accurately describe the quantity. Quantities that require both magnitude and direction are called vector quantities and can be represented by directional segments. These directional segments are called vectors. To help explain the difference let’s consider the following statements:

Mike was driving his motorcycle at 30 miles per hour down the street.

Mike was driving his motorcycle at 30 miles per hour heading east down the street.

The first statement is an example of a scalar quantity (speed) because we do not know which direction Mike is traveling. The second statement is an example of a vector quantity (velocity) because we know how fast Mike is travelling and which direction he is traveling in.

Applications of Vectors

Let’s take a look at a classic example of using vectors to illustrate a problem. Suppose two people are pulling on a rope in opposite directions (tug-o-war). Consider the following three diagrams and determine which direction the point A will travel:

Using vectors corresponding to the magnitude of the force be applied, we can visualize the direction of travel as a result of the forces being applied. If we knew the magnitude of the force being applied, we could calculate the resulting force as well. The resulting magnitude is easy to determine if the forces are being applied in the same or opposite directions. However, what if the forces are being applied at different angles such as:

To find the result of a situation similar to this, each vector needs to be broken down into its horizontal and vertical components.

Finding the Magnitude of the Horizontal and Vertical Components of a Vector

To find the magnitude of the horizontal, x, and vertical, y, components of a vector, we need to create a right triangle out of the vector as shown and apply some trigonometry:

$$\cos \theta = \frac{x}{r} \rightarrow x = r \cos \theta$$

$$\sin \theta = \frac{y}{r} \rightarrow y = r \sin \theta$$
When finding the magnitude of the horizontal and vertical components of a vector, please note the magnitude must be a positive number. The magnitude must be positive because the magnitude is a scalar value and does not refer to the direction of travel, so all values must be positive.

Magnitude of the horizontal component, $x = |r \cos \theta|$

Magnitude of the vertical component, $y = |r \sin \theta|$

Let’s try a few examples to help clarify everything.

**Example 1:** Find the magnitude of the horizontal and vertical components for the vector with magnitude of 150 and direction angle 130°.

Draw the vector and create a right triangle.

Find the magnitude of the horizontal component.

$$x = |150 \cos 50|$$

$$x \approx 96.4$$

Find the magnitude of the vertical component.

$$y = |150 \sin 50|$$

$$y \approx 114.9$$

**Note:** We found the magnitude of the components using the reference angle rather than the direction angle. The reason for using the reference angle rather than the direction angle is because if we use the reference angle the magnitudes will always be positive values.

**Example 2:** Find the magnitude of the horizontal and vertical components for the vector with magnitude of 86 and direction angle 218.5°.

Draw the vector and create a right triangle.

The reference angle for a 218.5° is 38.5°.
Find the magnitude of the horizontal component.

\[ x = 86 \cos 38.5 \]
\[ x \approx 67.3 \]

Find the magnitude of the vertical component.

\[ y = 86 \sin 38.5 \]
\[ y \approx 53.5 \]

**Example 3:** Find the magnitude of the horizontal and vertical components for the vector with magnitude of 320 and direction angle 305.2°.

Draw the vector and create a right triangle.

\[ \text{The reference angle for a 305.2° is 54.8°.} \]

Find the magnitude of the horizontal component.

\[ x = 320 \cos 54.8 \]
\[ x \approx 184.5 \]

Find the magnitude of the vertical component.

\[ y = 320 \sin 54.8 \]
\[ y \approx 261.5 \]